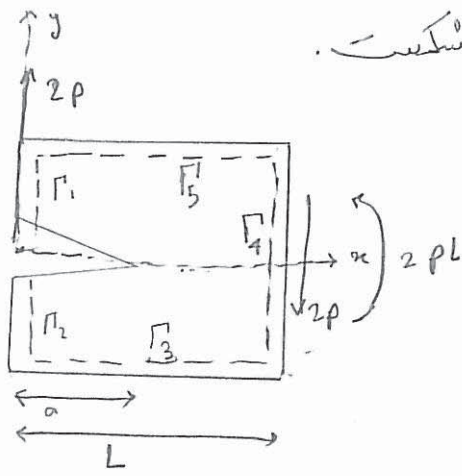


① تکلیف سری یوتا ۱۳ - سوحد محمدی ۹۸، ۲، ۹ - مکانیک شکست



$$J = J_{\Gamma_1} + J_{\Gamma_2} + J_{\Gamma_3} + J_{\Gamma_4} + J_{\Gamma_5}$$

$$J_{\Gamma_2} = J_{\Gamma_3} = J_{\Gamma_5} = 0$$

$$J_{\Gamma_4} = \int_{-h}^h [w dy - (\sigma_{\text{crack}} \frac{\delta u_{\text{crack}}}{\delta a} dy)]$$

$$\begin{cases} \frac{\delta u_{\text{crack}}}{\delta a} = \epsilon_{\text{crack}} \\ \frac{1}{2} \epsilon_{\text{crack}} \sigma_{\text{crack}} = w \end{cases}$$

$$= \int_{-h}^h [w dy - 2w dy] = - \int_{-h}^h w dy$$

$$w = \frac{\sigma_{\text{crack}}^2}{2E} \quad ; \quad \sigma_{\text{crack}} = \frac{M y}{I} = \frac{2 \rho L y}{I} \quad ; \quad I = \frac{B(2h)^3}{12} = \frac{2 B h^3}{3}$$

$$w = \frac{2 \rho^2 L^2 y^2}{2 E I^2} = \frac{2 \rho^2 L^2 y^2}{E I^2} = \frac{18 \rho^2 L^2 y^2}{4 E B^2 h^6}$$

$$J_{\Gamma_4} = - \int_{-h}^h \frac{9 \rho^2 L^2}{2 E B^2 h^6} y^2 dy = - \left(\frac{9 \rho^2 L^2}{2 E B^2 h^6} \right) \times \frac{2 h^3}{3} = \frac{-3 \rho^2 L^2}{E B^2 h^3}$$

$$J_{\Gamma_4} = \frac{-3 \rho^2 L^2}{E B^2 h^3}$$

$$J_{\Gamma_1} = - \int_0^h T \frac{\delta v}{\delta a} dy \Rightarrow$$

$$\frac{\delta^2 v}{\delta a^2} = \frac{2 \rho a}{EI} \Rightarrow \frac{\delta v}{\delta a} = \frac{2 \rho a^2}{2 EI} + C \Rightarrow \left(\frac{\delta v}{\delta a} \right)_{x=L} = 0 = \frac{\rho L^2}{EI} + C$$

$$C = \frac{-\rho L^2}{EI} \Rightarrow \frac{\delta v}{\delta a} = \frac{\rho a^2}{EI} - \frac{\rho L^2}{EI} \Rightarrow \left(\frac{\delta v}{\delta a} \right)_{x=0} = \frac{-\rho L^2}{EI}$$

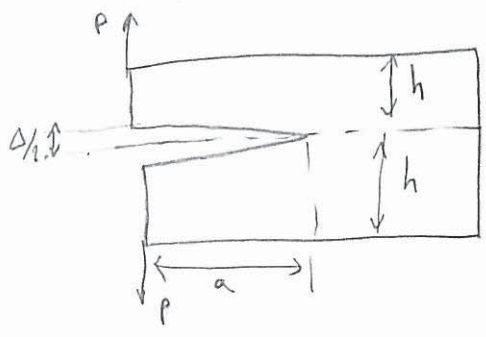
$$\frac{\delta V}{\delta u} \Big|_{u=0} = \frac{-PL^2}{EI}$$

$$I = \frac{Bh^3}{12} \Rightarrow \frac{\delta V}{\delta u} = \frac{-12PL^2}{EBh^3}$$

$$\Rightarrow J_{r_1} = - \int_0^h T \left(\frac{-12PL^2}{EBh^3} \right) dy = \frac{+24P^2L^2}{EBh^3}$$

$$J_{\sigma'} = \frac{+24P^2L^2}{EBh^3} - \frac{3P^2L^2}{EBh^3} = \frac{21P^2L^2}{EBh^3}$$

سلام عزیزان



$a = 10 \text{ cm}$
 $h = 1 \text{ cm}$

$E = 70 \text{ GPa}$

$\sigma_y = 500 \text{ MPa}$

$B = 2 \text{ mm}$

$r_y = 2 \text{ mm}$

نمره ۳۳

$P = P$

plane stress: $r_y = \frac{1}{2\pi} \left(\frac{k_1}{\sigma_{ys}} \right)^2$

$\Rightarrow \frac{\Delta}{2} = \frac{Pa^3}{3EI}$, $I = \frac{Bh^3}{12} \Rightarrow c = \frac{\Delta}{P} = \frac{2a^3}{3EI}$

$a = \frac{P^2}{2B} \cdot \frac{dc}{da} = \frac{P^2 a^2}{BEI} = \frac{12P^2 a^2}{B^2 h^3 E}$

$\Rightarrow G = \frac{k_1^2}{E^2} \Rightarrow k_1^2 = GE^2 = \frac{12P^2 a^2}{B^2 h^3}$

$r_y = \frac{1}{2\pi} \left(\frac{12P^2 a^2}{B^2 h^3 \sigma_{ys}^2} \right) \Rightarrow \sqrt{\frac{2\pi r_y B^2 h^3 \sigma_{ys}^2}{12 a^2}} = P$

$P = \sqrt{\frac{2\pi \times 2 \times 10^{-3} \times 4 \times 10^{-6} \times 10^{-6} \times 25 \times 10^{16}}{3 \times 12 \times 100 \times 10^{-4}}} = \sqrt{\frac{\pi}{3}} \times 10^5 \text{ Pa}$