بسمه تعالی

تمرین سری سوم مکانیک شکست

سام میرزاخانی: ۹۷۰۲۷۷۵

موعد تحویل ۹۷/۱۲/۲۲

1-

$$\cos\frac{\theta}{2}\left[K_I\sin\theta+K_{II}(3\cos\theta-1)\right]=0$$

$$\frac{d\sigma_{\theta\theta}}{d\theta}=0\Rightarrow\frac{3}{2}\frac{K_I}{\sqrt{2\pi r}}\left[-\frac{1}{4}\sin(\theta/2)-\frac{1}{4}\sin(3\theta/2)\right]+\frac{3}{2}\frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{1}{4}\cos(\theta/2)-\frac{3}{4}\cos(3\theta/2)\right]=0$$

$$\begin{cases}\sin 3\theta/2=3\sin\theta/2-4\sin^3\theta/2\\ \cos 3\theta/2=4\cos^3\theta/2-3\cos\theta/2\end{cases}$$

$$\Rightarrow K_I\left[-\frac{1}{4}\sin\theta/2-\frac{3}{4}\sin\theta/2+\sin^3\theta/2\right]+K_{II}\left[-\frac{1}{4}\cos\theta/2-3\cos^3\theta/2+\frac{9}{4}\cos\theta/2\right]=0$$

$$K_I\left[-\sin\theta/2+\sin^3\theta/2\right]+K_{II}\left[2\cos\theta/2-3\cos^3\theta/2\right]=0$$

$$-K_I\sin\theta/2\left[1-\sin^2\theta/2\right]+K_{II}\cos\theta/2\left[2-3\cos^2\theta/2\right]=0$$

$$-K_I\sin\theta/2\cdot\cos^2\theta/2+K_{II}\cos\theta/2\left[\frac{1}{2}-\frac{3}{2}\cos\theta\right]=0$$

$$-\frac{K_I}{\cancel{2}}\sin\theta\cdot\cos\theta/2-\frac{K_{II}}{\cancel{2}}\cos\theta/2\left[-1+3\cos\theta\right]=0$$

$$\cos\theta/2\left[K_I\sin\theta+K_{II}(3\cos\theta-1)\right]=0$$

Plot: $\dfrac{\sigma_{\theta\theta}\sqrt{2\pi r}}{K_I}$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right] + \frac{K_{II}}{\sqrt{2\pi r}}\left[-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\Rightarrow \sigma_{\theta\theta} \times \frac{\sqrt{2\pi r}}{K_I} : \left[\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right] + \frac{K_{II}}{K_I}\left[-\frac{3}{4}\sin\frac{\theta}{2} - \frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right] = F$$

$\dfrac{K_{II}}{K_I} = 0.5$ and 2

$\Rightarrow$ با رسم تابع F برای مقادیر مختلف $\dfrac{K_{II}}{K_I}$ می‌توان مقادیر ماکزیمم این تابع را برای حالت های مختلف بدست آورد.

که با استفاده از دستور متلب :

$$\frac{K_{II}}{K_I} = 0.5 \Rightarrow \boxed{\theta_{max} = -40.5}$$

با جایگذاری در رابطه :

$$\cos\frac{\theta}{2}\left[\sin\theta \cdot K_I + (3\cos\theta - 1)K_{II}\right] \cong 0 \quad (*)$$

که در رابطه فوق صدق می‌کند.

برای : $\dfrac{K_{II}}{K_I} = 2 \Rightarrow \boxed{\theta_{max} = -61.5}$

که باز هم در رابطه فوق صدق می‌کند.

9702775 سام میرزاخانی

3.)

$$G = \frac{k+1}{8\mu}\left[(C_{11}^2+C_{21}^2)K_I^2+(C_{12}^2+C_{22}^2)K_{II}^2+2(C_{11}C_{12}+C_{21}C_{22})K_I K_{II}\right]$$

$$C_{11}=\frac{1}{4}\left[3\cos\alpha/2+\cos\frac{3\alpha}{2}\right] \; ; \; C_{12}=-\frac{3}{4}\left[\sin\alpha/2+\sin 3\alpha/2\right]$$

$$C_{21}=\frac{1}{4}(\sin\alpha/2+\sin 3\alpha/2) \; ; \; C_{22}=\frac{1}{4}\left[\cos\alpha/2+3\cos 3\alpha/2\right]$$

For mod II : $K_I = 0$

$$\Rightarrow G = \frac{k+1}{8\mu}\left[(C_{12}^2+C_{22}^2)K_{II}^2\right]$$

$$G = \frac{k+1}{8\mu}\left[\frac{9}{16}\left(\sin^2\alpha/2+\sin^2 3\alpha/2+2\sin\alpha/2\cdot\sin 3\alpha/2\right)+\frac{1}{16}\left(\cos^2\alpha/2+9\cos 3\alpha/2+6\cos\alpha/2\cdot\cos 3\alpha/2\right)\right]$$

$$\frac{dG}{d\alpha}=0 \Rightarrow \frac{9}{\cancel{16}}\left[\sin\alpha/2\cdot\cos\alpha/2+3\sin 3\alpha/2\cdot\cos 3\alpha/2+\cos\alpha/2\cdot\sin 3\alpha/2+3\sin\alpha/2\cdot\cos 3\alpha/2\right]\cdots$$

$$+\frac{1}{\cancel{16}}\left[-\cos\alpha/2\cdot\sin\alpha/2-27\cos 3\alpha/2\cdot\sin 3\alpha/2-3\sin\alpha/2\cdot\cos 3\alpha/2-9\cos\alpha/2\cdot\sin 3\alpha/2\right]=0$$

$$\Rightarrow \cancel{8}\sin\alpha/2\cdot\cos\alpha/2+\overset{3}{\cancel{24}}\sin\alpha/2\cdot\cos 3\alpha/2=0$$

$$=\frac{1}{2}\sin\alpha+\frac{3}{2}(\sin 2\alpha-\sin\alpha)=0 \Rightarrow 6\cos\alpha-2=0 \Rightarrow$$

$$\Rightarrow \cos\alpha=\frac{1}{3} \Rightarrow \boxed{\alpha=70.6^\circ}$$

For mod I : $K_{II} = 0$

$$G = \frac{k+1}{8\mu}\left[(C_{11}^2 + C_{21}^2) K_I^2 \right]$$

$$G = \frac{k+1}{8\mu} \times \frac{1}{16}\left[\left(3\cos\alpha/2 + \cos 3\alpha/2\right)^2 + \left(\sin\alpha/2 + \sin 3\alpha/2\right)^2 \right] K_I^2$$

$$\frac{dG}{d\alpha} = 0 \Rightarrow \left[-9\sin\alpha/2 \cdot \cos\alpha/2 - \cancel{3\cos 3\alpha/2 \cdot \sin 3\alpha/2} - \cancel{3\sin\alpha/2 \cdot \cos 3\alpha/2} - 9\cos\alpha/2 \cdot \sin 3\alpha/2 \right]$$

$$\cdots + \left[\cos\alpha/2 \cdot \sin\alpha/2 + \cancel{3\cos\frac{3\alpha}{2} \cdot \sin\frac{3\alpha}{2}} + \cos\alpha/2 \cdot \sin 3\alpha/2 + \cancel{3\sin\alpha/2 \cdot \cos\frac{3\alpha}{2}} \right]$$

$$\Rightarrow -\cancel{8}\sin\alpha/2 \cdot \cos\alpha/2 - \cancel{8}\cos\alpha/2 \cdot \sin 3\alpha/2 = 0$$

$$\Rightarrow \cos\alpha/2\left[\sin\alpha/2 + \sin 3\alpha/2\right] = 0$$

$$\sin\alpha/2 + \sin 3\alpha/2 = 2\sin\alpha \cdot \cos\alpha/2$$

$$\Rightarrow \cos^2\alpha/2\left[2\sin\alpha\right] = 0$$

$$\Rightarrow \sin\alpha = 0 \Rightarrow \boxed{\alpha = 0}$$

« سام سوزافن 9702775 »

تکمیل سوال I