تکلیف سری سوم مکانیک شکست   موعد تحویل: سه‌شنبه ۹۷/۱۱/۲۱ در کلاس

1- Prove that Eq. $\cos\theta/2[K_I \sin\theta + K_{II}(3\cos\theta - 1)] = 0$ can be obtained from $\partial\sigma_{\theta\theta}/\partial\theta = 0$.

2- Plot $\sigma_{\theta\theta}\sqrt{2\pi r}/K_I$ versus θ for K_{II}/K_I = 0.5 and 2.0, respectively. Verify that Eq. $\cos\theta/2[K_I \sin\theta + K_{II}(3\cos\theta - 1)] = 0$ gives the orientation at which $\sigma_{\theta\theta}$ reaches the maximum.

3- In mixed mode conditions, the energy release rate at the kink tip is (as shown in following figure):

$$G = \frac{\kappa+1}{8\mu}\left[\left(C_{11}^2 + C_{21}^2\right)K_I{}^2 + \left(C_{12}^2 + C_{22}^2\right)K_{II}{}^2 + 2\left(C_{11}C_{12} + C_{21}C_{22}\right)K_I K_{II}\right]$$

where:

$$C_{11} = \frac{1}{4}\left(3\cos\frac{\alpha}{2} + \cos\frac{3\alpha}{2}\right)$$
$$C_{12} = -\frac{3}{4}\left(\sin\frac{\alpha}{2} + \sin\frac{3\alpha}{2}\right)$$
$$C_{21} = \frac{1}{4}\left(\sin\frac{\alpha}{2} + \sin\frac{3\alpha}{2}\right)$$
$$C_{22} = \frac{1}{4}\left(\cos\frac{\alpha}{2} + 3\cos\frac{3\alpha}{2}\right)$$

The crack growth direction, or fracture angle α_0, is thus determined by maximizing $G(\alpha)$:

$$\frac{\partial G(\alpha)}{\partial\alpha} = 0 \qquad \text{at} \quad \alpha = \alpha_0 \qquad (*)$$
$$\frac{\partial^2 G(\alpha)}{\partial\alpha^2} < 0 \qquad \text{at} \quad \alpha = \alpha_0$$

Show that the fracture angles determined from Eq. (*) are the same as that predicted from the maximum hoop stress criterion (especially for Mode I and II).