1- A steel strap 1-mm thick and 20-mm wide with a through-thickness center crack 4 mm long is loaded to failure. (a) Determine the critical load if K_{IC} = 80 MPa.m$^{1/2}$ for the strap material. (b) Use an available correction factor, f (a/w), for this crack configuration and calculate the critical stress as σ_c.

Solution:

a)

$$B = 1mm,\ 2W = 20mm,\ 2a = 4\ mm, a/2W = 0.10,\ K_{IC} = 80\ MPa\sqrt{m}$$

$$\frac{a}{2W} = 0.1\ rad = \frac{0.1\times 180}{\pi} = 5.73^{\circ}$$

$$k_I = \sigma\sqrt{\pi a}\left[\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)\right]^{\frac{1}{2}} \Rightarrow f\left(\frac{a}{w}\right) = \sqrt{\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)} = \sqrt{\frac{20}{\pi\times 2}\tan\left(\pi\times 5.73\right)} = 1.0169$$

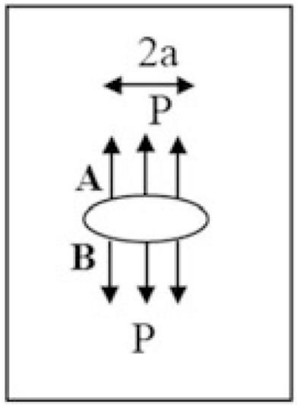
$$\sigma = \frac{k_{Ic}}{1.0169\sqrt{\pi a}} = \frac{80\times 10^6}{1.0169\sqrt{2\pi\times 10^{-3}}} = 992.38\,MPa$$

b)

$$k_I = \sigma\sqrt{\pi a}\ f\left(\frac{a}{w}\right) \Rightarrow f\left(\frac{a}{w}\right) = \sqrt{\sec\frac{\pi a}{2W}}\left[1-0.025\left(\frac{a}{w}\right)^2 + 0.06\left(\frac{a}{w}\right)^4\right] = 0.8$$

$$\sigma = \frac{k_{Ic}}{0.8\sqrt{\pi a}} = \frac{80\times 10^6}{0.8\sqrt{2\pi\times 10^{-3}}} = 1261.43MPa$$

2- The plate below has an internal crack subjected to a pressure P on the crack surface. The stress intensity factors at points A and B are



$$K_A = \int \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a+x}{a-x}} \cdot dx$$

$$K_B = \int \frac{P}{\sqrt{\pi a}} \sqrt{\frac{a-x}{a+x}} \cdot dx$$

Use the principle of superposition to show that the total stress intensity factor is defined by $K_I = P\sqrt{\pi a}$.

Solution:

$$\left.\begin{aligned} k_{\mathrm{I}} = k_A + k_B = \frac{P}{\sqrt{\pi a}} \int \left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}} \right) dx \\ \sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}} = \frac{(a+x)+(a-x)}{\sqrt{(a-x)(a+x)}} = \frac{2a}{\sqrt{a^2-x^2}} \end{aligned}\right\} \Rightarrow k_{\mathrm{I}} = 2P\sqrt{\frac{a}{\pi}} \int \frac{dx}{\sqrt{a^2-x^2}} = 2P\sqrt{\frac{a}{\pi}} \arcsin\frac{x}{a}$$

$$for \;\; x = a \rightarrow \arcsin(1) = \frac{\pi}{2} \Rightarrow k_{\mathrm{I}} = 2P\sqrt{\frac{a}{\pi}} \times \frac{\pi}{2} = P\sqrt{\pi a}$$

3- A material exhibits the following crack growth resistance behavior: $R = 6.95\sqrt{a - a_0}$

where a_0 is the initial crack size. R has units of kJ/m^2 and the crack size is in mm.

The elastic modulus of this material 207,000 MPa. Consider a wide plate with a through crack ($a << W$) that is made from this material.

a. If this plate fractures at 138 MPa, compute the following:

i. The half crack size at failure (a_c).

ii. The amount of stable crack growth (at each crack tip) that precedes failure ($a_c - a_0$).

b. If this plate has an initial crack length ($2a_o$) of 50.8 mm and the plate is loaded to failure, compute the following:

i. The stress at failure.

ii. The half crack size at failure.

iii. The stable crack growth at each crack tip.

Solution:

$$\Pi = U_0 - \frac{\pi\sigma^2 a^2 B}{E} \Rightarrow G = -\frac{d\Pi}{dA}\ ,\ A = 2aB \Rightarrow G = \frac{\pi\sigma^2 a}{E}$$

a)

$$\left.\begin{array}{l} G = R \\ \dfrac{dG}{da} = \dfrac{dR}{da} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \dfrac{\pi\sigma^2 a_c}{E} = 6.65\sqrt{a_c - a_0} \\ \dfrac{\pi\sigma^2}{E} = 3.475\left(a_c - a_0\right)^{-0.5} \end{array}\right.$$

$$\frac{\pi\left(138\times10^6\right)^2}{207000\times10^6} = 3.475\left(a_c - a_0\right)^{-0.5} \Rightarrow a_c - a_0 = 145mm$$

$$\frac{\pi\left(138\times10^6\right)^2 a_c}{207000\times10^6} = 6.65\left(145\right)^{0.5} \Rightarrow a_c = 290mm\ ,\ a_0 = 145mm$$

b)

$$\frac{\pi\sigma^2}{E} = 3.475\left(a_c - a_0\right)^{-0.5} \rightarrow \sigma^2 = \frac{3.475E\left(a_c - a_0\right)^{-0.5}}{\pi} \Rightarrow \frac{\pi\sigma^2 a_c}{E} = 6.65\sqrt{a_c - a_0} \rightarrow a_c = 2\left(a_c - a_0\right)$$

$$a_0 = 25.4mm\ , a_c = 50.8mm$$

$$\frac{0.0508\pi\sigma^2}{207000\times10^6} = 6.65\sqrt{0.0254} \rightarrow \sigma = 213MPa$$